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GAIN SCHEDULING CONTROL OF GAS TURBINE ENGINES: STABILITY BY COMPUTING A SINGLE QUADRATIC LYAPUNOV FUNCTION

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ABSTRACT

We develop and describe a stable gain scheduling controller for a gas turbine engine that drives a variable pitch propeller. A stability proof is developed for gain scheduled closed-loop system using global linearization and linear matrix inequality (LMI) techniques. Using convex optimization tools, a single quadratic Lyapunov function is computed for multiple linearizations near equilibrium and non-equilibrium points of the nonlinear closed-loop system. This approach guarantees stability of the closed-loop gas turbine engine system. Simulation results show the developed gain scheduling controller is capable of regulating a turboshaft engine for large thrust commands in a stable fashion with proper tracking performance.

NOMENCLATURE

N_1 Low Spool Speed (Non-dimensional)

N_2 High Spool Speed (Non-dimensional)
u_1 Fuel Flow Control Input (Non-dimensional)
u_2 Propeller Pitch Angle Control Input (Deg)
T Thrust (N)
TSFC   Thrust Specific Fuel Consumption (kg/s/N)
α Scheduling Parameter
λ Eigenvalue

1   Introduction

Stability and control of gas turbine engines have been of interest to researchers and engineers from a variety of perspectives. An introduction to the analysis and design of engine control systems can be found in [1]. The basics of controlling a gas turbine engine while satisfying numerous constraints has been reviewed in [2]. The design of engine control and monitoring systems with a dual interest in both turbofan and turboshaft engines has been covered in [3]. An application of robust stability analysis tools

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We develop and describe a stable gain scheduling controller for a gas turbine engine that drives a variable pitch propeller. A stability proof is developed for gain scheduled closed-loop system using global linearization and linear matrix inequality (LMI) techniques. Using convex optimization tools, a single quadratic Lyapunov function is computed for multiple linearizations near equilibrium and non-equilibrium points of the nonlinear closedloop system. This approach guarantees stability of the closedloop gas turbine engine system. Simulation results show the developed gain scheduling controller is capable of regulating a turboshaft engine for large thrust commands in a stable fashion with proper tracking performance.

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for uncertain turbine engine systems is presented in [4]. An application of the Linear-Quadratic-Gaussian with Loop-Transfer-Recovery methodology to design a control system for the F-100 turbofan engine is presented in [5], and for a simplified turbofan engine model is considered in [6]. A unified robust multivariable approach to propulsion control design has been developed in [7]. The development of other control techniques, such as sliding mode, for gas turbine engine application can be found in [8].

To facilitate the stability analysis of nonlinear systems, such as gas turbine engines, an efficient technique is to approximate them by a linear time-varying (LTV) system. This concept, which is known as global linearization, can be found in [9, 10]. More recent work on global linearization and the use of Linear Matrix Inequalities (LMIs) for the analysis of dynamical systems can be found in [11]. Some Soviet literatures on the absolute stability problem, like Lur'e and Postnikov [12, 13] and Popov [14–16], also implicitly use the idea of global linearization. Recent literatures that demonstrate the practical power of global linearization technique include [17, 18], and [19]. [19] uses this idea along with the notion of incremental stability.

One of the control design approaches, which perhaps is one of the most popular nonlinear control design approaches and has been widely and successfully applied in fields ranging from aerospace to process control [20, 21], is gain scheduling. Gas turbine engines are no exception, and research on gain scheduling control of gas turbine engines is presented in [22–29]. A simplified scheme for scheduling multivariable controllers for robust performance over a wide range of turbofan engine operating points is presented in [23]. In a recent work presented in [27], results on polynomial fixed-order controller design are extended to SISO gain-scheduling with guaranteed stability and H_∞ performance for a turbofan engine, over the whole scheduling parameter range. In this work, the engine Linear Parameter Varying (LPV) representation depends on an exogenous variable parameter which is the combustion chamber pressure.

In the previous works [30, 31] the authors discussed controllers for single spool and twin spool turboshaft systems. Those controllers are designed for small transients, and small throttle commands. In this paper we develop an output dependent gain scheduled control structure for a MIMO linear parameter dependent model of a JetCat SPT5 turboshaft engine [32] using the method presented in [20,33–35]. This controller is designed to be used for the entire flight envelope of the twin spool turboshaft engine with stability guarantees. The scheduling variable in our design process is an *endogenous* parameter, which is a function of the gas turbine engine spool speeds. This endogenous scheduling variable captures the plant nonlinearities, as explained in [33,34], since the spool speeds are the main states of the turboshaft engine state-space model, and also the outputs of the system. The stability analysis for the closed-loop system presented, and the essential part of the stability analysis is to find a common quadratic

Lyapunov function for multiple linearizations near equilibrium and non-equilibrium points, which are distributed over the entire operational envelope of the plant.

The paper is organized as follows: First, a linear parameter dependent representation of the plant is presented. Concepts for output dependent gain scheduled control of this model are then developed. Third, the stability analysis of the closed-loop system is presented. Finally, simulation results for gain scheduling control of a MIMO physics-based model of a JetCat SPT5 turboshaft engine are presented. Simulation results show the successful application of the proposed controller for the entire flight envelope of the turboshaft engine with guaranteed stability and proper tracking performance.

2 Gain Scheduling Control Design

Consider the nonlinear dynamical system

$$
\begin{aligned}
\dot{x}^p &= f^p(x^p, u), \\
y &= g^p(x^p, u),
\end{aligned}
\tag{1}
$$

where $x^p \in \mathfrak{R}^n$ is the state vector, $u \in \mathfrak{R}^m$ is the control input vector, $y \in \mathfrak{R}^m$ is the output vector, $f^p(.)$ is an n-dimensional differentiable nonlinear vector function which represents the plant dynamics, and $g^p(.)$ is an m-dimensional differentiable nonlinear vector function which generates the plant outputs. We intend to design a feedback control such that y properly tracks a reference signal r as time goes to infinity, where $r \in D_r \subset \mathfrak{R}^m$, and D_r is a compact set.

Assume that for each $r \in D_r$, there is a unique pair (x_e^p, u_e) that depends continuously on r and satisfies the equations

$$
\begin{aligned}
0 &= f^p(x_e^p, u_e), \\
r &= g^p(x_e^p, u_e),
\end{aligned}
\tag{2}
$$

x_e^p is the desired equilibrium point and u_e is the steady-state control that is needed to maintain equilibrium at x_e^p. It is often useful to parameterize the family of system equilibria as follows:

Definition 1. *The functions $x_e^p(\alpha), u_e(\alpha)$, and $r_e(\alpha)$ define an equilibrium family for the plant (1) on the set Ω if*

$$
\begin{aligned}
f^p(x_e^p(\alpha), u_e(\alpha)) &= 0, \\
g^p(x_e^p(\alpha), u_e(\alpha)) &= r_e(\alpha), \ \alpha \in \Omega.
\end{aligned}
\tag{3}
$$

Let $O \subset \mathfrak{R}^{m+n}$ be the region of interest for all possible system state and control vector (x^p, u) during the system operation,

and denote x_{ei}^p and u_{ei}, $i \in I = 1, 2, ..., q$, as a set of constant operating points located at some representative and properly separated points inside O. Introduce a set of q regions $O_i, i \in I$ centered at the chosen operating points (x_{ei}^p, u_{ei}), and denote their interiors as O_{i0}, such that $O_{j0} \bigcap O_{k0} = \oslash$ for all $j \neq k$, and $\bigcup_{i=1}^{l} O_i = O$. The linearization of the plant at each equilibrium point is

$$
\begin{aligned}
\dot{x}^p &= A_i^p(x^p - x_{ei}^p) + B_i^p(u - u_{ei}), \\
y &= C_i^p(x^p - x_{ei}^p) + D_i^p(u - u_{ei}) + y_{ei},
\end{aligned}
\tag{4}
$$

where the matrices are obtained as follows

$$
\begin{aligned}
A_i^p &= \frac{\partial f^p}{\partial x^p}\Big|_{(x_{ei}^p, u_{ei})}, \quad \forall (x^p, u) \in O_i, \\
B_i^p &= \frac{\partial f^p}{\partial u}\Big|_{(x_{ei}^p, u_{ei})}, \quad \forall (x^p, u) \in O_i, \\
C_i^p &= \frac{\partial g^p}{\partial x^p}\Big|_{(x_{ei}^p, u_{ei})}, \quad \forall (x^p, u) \in O_i, \\
D_i^p &= \frac{\partial g^p}{\partial u}\Big|_{(x_{ei}^p, u_{ei})}, \quad \forall (x^p, u) \in O_i.
\end{aligned}
\tag{5}
$$

Note that (x^p, u) belongs to only one O_i at each time. Corresponding to each linearization at ith equilibrium point, there exist an $\alpha_i \in \Omega$, which is a function of equilibrium values of the system outputs, i.e. y_{ei}.

The family of plant linear models (4) can be written as

$$
\begin{aligned}
\delta \dot{x}^p &= A^p(\alpha)\delta x^p + B^p(\alpha)\delta u, \\
\delta y &= C^p(\alpha)\delta x^p + D^p(\alpha)\delta u, \quad \forall \alpha \in \Omega,
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
\delta x^p &= x^p - x_e^p(\alpha) \\
\delta y &= y - y_e(\alpha), \\
\delta u &= u - u_e(\alpha).
\end{aligned}
\tag{7}
$$

$A^p(\alpha)$, $B^p(\alpha)$, $C^p(\alpha)$, and $D^p(\alpha)$ are the parameterized plant linearization family matrices and $x_e^p(\alpha)$, $u_e(\alpha)$, and $y_e(\alpha)$ are the parameterized steady-state variables for the states, inputs and outputs of the plant, which form the equilibrium manifold of plant (1). The subscript "e" stands for "steady-state" throughout this paper.

Based on the results from [20, 33–35], an output dependent gain scheduled controller for plant (6) is designed as follows. First, a set of parameter values α_i is selected, which represent the range of the plant's dynamics, and a linear time-invariant controller is designed for each corresponding linear

model. Then, in between operating points, the controller gains are linearly interpolated such that for all frozen values of the parameters, the closed-loop system has satisfactory properties, such as nominal stability and robust performance. To guarantee that the closed-loop system retains the dynamic properties of the frozen-parameter designs, the scheduling variables should vary slowly with respect to the system dynamics [33]. Figure 1, shows schematically how the output dependent gain scheduled controller works.
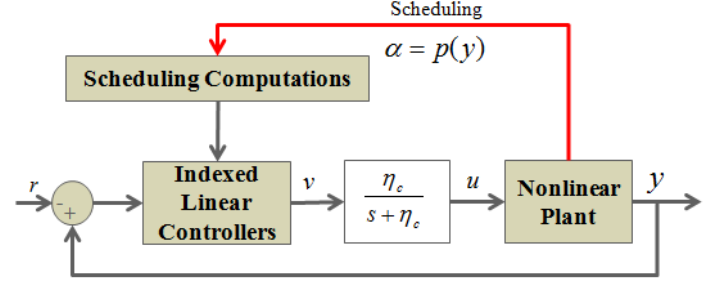

FIGURE 1. Output dependent gain scheduling controller diagram

The parameter α is called the scheduling variable and should be measurable in real time. α can be a function of endogenous variables (i.e., depending on the plant states) and/or exogenous variables (i.e., independent of the plant states). In LPV systems, this parameter is an exogenous parameter [36]. Some of the examples of exogenous parameter selection in LPV control of turbine engines are presented in [25–27]. In [25], the scheduling parameter is defined as a function of the exogenous signals describing the surroundings, like altitude, intake Mach number, and a health parameter describing the state of the compressor. In [26], the scheduling parameter is defined as a function of lagged measurement of engine thrust and altitude, which are exogenous variables. In [27], the scheduling parameter is defined to be the combustion chamber pressure, which is an exogenous variable. In gain-scheduling, this parameter is a function of the output and hence it is an endogenous parameter [36]. Some of the examples of endogenous parameter selection for gain-scheduled control of turbine engines can be found in [22, 28, 29]. In [22], the scheduling parameter is defined to be the engine low pressure spool speed, which is one of the outputs of the system. In [28, 29], the scheduling parameter is defined to be the engine high pressure spool speed. In the turboshaft engine control example described later in this paper, α is defined to be the Euclidean norm of the engine spool speeds, which can be measured in real-time. Since the spool speeds are the only plant states in the model and also due to the fact that we need the plant nonlinearities to be captured by the output vector, as explained in [33, 34], we defined the scheduling parameter to be the function of both spool speeds.

3

On the other hand, for a simpler interpolation process, we wanted the scheduling parameter to be a scalar, so we used the Euclidean norm of the output vector.

The design of a linearization gain scheduled controller requires designing a linear controller family corresponding to the plant linearization family (6). Let the parameterized linear controller family be

$$\delta \dot{x}^c = A^c(\alpha)\delta x^c + B^c(\alpha)[\delta y - \delta r],$$
$$\delta u = C^c(\alpha)\delta x^c + D^c(\alpha)[\delta y - \delta r], \quad \forall \alpha \in \Omega, \tag{8}$$

where

$$\delta x^c = x^c - x_e^c(\alpha),$$
$$\delta r = r - r_e(\alpha), \quad \forall \alpha \in \Omega. \tag{9}$$

$x_e^c(\alpha)$ and $r_e(\alpha)$ are the parameterized steady-state variables for the controller states and reference signals. A standard realization of the parameterized controller can be written with the reference signal explicitly displayed as

$$\begin{bmatrix} \delta \dot{x}^c \\ \delta u \end{bmatrix} = \begin{bmatrix} A^c(\alpha) & B^c(\alpha) & -B^c(\alpha) \\ C^c(\alpha) & D^c(\alpha) & -D^c(\alpha) \end{bmatrix} \begin{bmatrix} \delta x^c \\ \delta y \\ \delta r \end{bmatrix}, \quad \forall \alpha \in \Omega. \tag{10}$$

We have to obtain, based on the linear controller family (10), a controller that has the general form

$$\dot{x}^c = f^c(x^c, y, r),$$
$$u = g^c(x^c, y, r), \tag{11}$$

with the input and output signals corresponding to the nonlinear plant (1). $f^c(.)$ is an m-dimensional differentiable nonlinear vector function which represents the controller dynamics, and $g^c(.)$ is an m-dimensional differentiable nonlinear vector function which generates the controller outputs.

The objective in linearization scheduling is that the equilibrium family of the controller (11) match the plant equilibrium family, so that the closed-loop system maintains suitable trim values, and the linearization family of the controller obtained from linearizing (11) is the same as the designed family of linear controllers shown in (8) [20]. For the equilibrium conditions of plant (1) and controller (11) to match, there must exist a function $x_e^c(\alpha)$ such that

$$0 = f^c(x_e^c(\alpha), y_e(\alpha), r_e(\alpha)),$$
$$u_e(\alpha) = g^c(x_e^c(\alpha), y_e(\alpha), r_e(\alpha)), \quad \forall \alpha \in \Omega, \tag{12}$$

where

$$A^c(\alpha) = \frac{\partial f^c}{\partial x^c}\big|_{(x_e^c(\alpha), y_e(\alpha), r_e(\alpha))},$$
$$B^c(\alpha) = \frac{\partial f^c}{\partial y}\big|_{(x_e^c(\alpha), y_e(\alpha), r_e(\alpha))},$$
$$C^c(\alpha) = \frac{\partial g^c}{\partial x^c}\big|_{(x_e^c(\alpha), y_e(\alpha), r_e(\alpha))}, \tag{13}$$
$$D^c(\alpha) = \frac{\partial g^c}{\partial y}\big|_{(x_e^c(\alpha), y_e(\alpha), r_e(\alpha))}, \quad \forall \alpha \in \Omega.$$

So the controller family has the form

$$\dot{x}^c = A^c(\alpha)[x^c - x_e^c(\alpha)] + B^c(\alpha)[y - r],$$
$$u = C^c(\alpha)[x^c - x_e^c(\alpha)] + D^c(\alpha)[y - r] + u_e(\alpha), \quad \forall \alpha \in \Omega. \tag{14}$$

Note that $r_e(\alpha) = y_e(\alpha)$, as a result $\delta y - \delta r = y - r$. The scheduling parameter α is treated as a parameter throughout the design process, and then it becomes a time-varying input signal to the gain-scheduled controller implementation through the dependence $\alpha(t) = p(y(t))$. The parameter α is an endogenous variable, since it is a function of the plant outputs. Replacing α with $p(y)$, the gain scheduled controller becomes

$$\dot{x}^c = A^c(p(y))[x^c - x_e^c(p(y))] + B^c(p(y))[y - r],$$
$$u = C^c(p(y))[x^c - x_e^c(p(y))] + D^c(p(y))[y - r] + u_e(p(y)). \tag{15}$$

Linearization of (15) about an equilibrium specified by α yields

$$\delta \dot{x}^c = A^c(\alpha)\delta x^c + B^c(\alpha)[y - r]$$
$$\quad - [A^c(\alpha)\frac{\partial x_e^c(\alpha)}{\partial \alpha}] \times [\frac{\partial p}{\partial y}(y_e(\alpha))\delta y],$$
$$\delta u = C^c(\alpha)\delta x^c + D^c(\alpha)[y - r] \tag{16}$$
$$\quad + [\frac{\partial u_e(\alpha)}{\partial \alpha} - C^c(\alpha)\frac{\partial x_e^c(\alpha)}{\partial \alpha}] \times [\frac{\partial p}{\partial y}(y_e(\alpha))\delta y].$$

Comparing (16) with (10), we see there are additional terms, and we refer to them as hidden coupling terms following the notation of [20]. In order to get rid of these hidden coupling terms, the following condition must be satisfied

$$[A^c(\alpha)\frac{\partial x_e^c(\alpha)}{\partial \alpha}] \times [\frac{\partial p}{\partial y}(y_e(\alpha))\delta y] = 0,$$
$$[\frac{\partial u_e(\alpha)}{\partial \alpha} - C^c(\alpha)\frac{\partial x_e^c(\alpha)}{\partial \alpha}] \times [\frac{\partial p}{\partial y}(y_e(\alpha))\delta y] = 0. \tag{17}$$

It is not always easy to come up with solutions to satisfy condition (17). In order to make the design process easier, we control

the system via filtered inputs, rather than the input themselves, so there is no need for equilibrium control value other than zero (i.e. $x_e^c(\alpha) = 0, v_e(\alpha) = 0, \forall \alpha$, where $v_e(\alpha)$ is the parameterized steady-state variables for the new inputs).

The plant (1) with the filtered inputs, becomes

$$\begin{bmatrix} \dot{x}^p \\ \dot{u} \end{bmatrix} = \begin{bmatrix} f^p(x^p, u) \\ -\eta_c u \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_c \times I \end{bmatrix} v,$$
$$y = g^p(x^p, u), \tag{18}$$

The controller has the general form

$$\dot{x}^c = f^c(x^c, y, r),$$
$$v = g^c(x^c, y, r), \tag{19}$$

with the input and output signals corresponding to those of the nonlinear plant (18). Now, combining (18) and (19) leads to

$$\begin{bmatrix} \dot{x}^p \\ \dot{u} \\ \dot{x}^c \end{bmatrix} = \underbrace{\begin{bmatrix} f^p(x^p, u) \\ -\eta_c u \\ f^c(x^c, g^p(x^p, u), r) \end{bmatrix}}_{f(x,r)} + \underbrace{\begin{bmatrix} 0 \\ \eta_c \times I \\ 0 \end{bmatrix}}_{B} v,$$
$$v = \underbrace{g^c(x^c, g^p(x^p, u), r)}_{g(x,r)}, \tag{20}$$

and the closed-loop nonlinear system is

$$\dot{x} = f(x, r) + Bg(x, r),$$
$$= F(x, r), \tag{21}$$

where $x \in D_x \subset \mathfrak{R}^{n+2m}$, and $r \in D_r \subset \mathfrak{R}^m$. The augmented linear family of systems for the augmented plant (18) becomes

$$\underbrace{\begin{bmatrix} \delta \dot{x}^p \\ \delta \dot{u} \end{bmatrix}}_{\delta \dot{x}_{aug}} = \underbrace{\begin{bmatrix} A^p(\alpha) & B^p(\alpha) \\ 0 & -\eta_c \times I \end{bmatrix}}_{A_{aug}(\alpha)} \underbrace{\begin{bmatrix} \delta x^p \\ \delta u \end{bmatrix}}_{\delta x_{aug}} + \underbrace{\begin{bmatrix} 0 \\ \eta_c \times I \end{bmatrix}}_{B_{aug}} \delta v,$$
$$\delta y = \underbrace{[C^p(\alpha), D^p(\alpha)]}_{C_{aug}(\alpha)} \underbrace{\begin{bmatrix} \delta x^p \\ \delta u \end{bmatrix}}_{\delta x_{aug}}, \tag{22}$$

and the controller has the similar structure as (8)

$$\delta \dot{x}^c = A_v^c(\alpha) \delta x^c + B_v^c(\alpha)[\delta y - \delta r],$$
$$\delta v = C_v^c(\alpha) \delta x^c + D_v^c(\alpha)[\delta y - \delta r], \quad \forall \alpha \in \Omega, \tag{23}$$

where

$$\delta v = v - v_e(\alpha), \quad \forall \alpha \in \Omega. \tag{24}$$

Now, since $x_e^c(\alpha) = 0$, $v_e(\alpha) = 0$, $\forall \alpha$, the controller is

$$\dot{x}^c = A_v^c(\alpha) x^c + B_v^c(\alpha)[\delta y - \delta r],$$
$$v = C_v^c(\alpha) x^c + D_v^c(\alpha)[\delta y - \delta r], \quad \forall \alpha \in \Omega, \tag{25}$$

rewriting controller (25) with $\alpha(t) = p(y(t))$, we obtain

$$\dot{x}^c = A_v^c(p(y)) x^c + B_v^c(p(y))[y - r],$$
$$v = C_v^c(p(y)) x^c + D_v^c(p(y))[y - r]. \tag{26}$$

Linearization of (26) about an equilibrium specified by α gives (25), so there are no hidden coupling terms similar to the ones we saw in (16), and the condition (17) is satisfied. One of the options for control design is to set the controller matrices as follows

$$A_v^c(\alpha) = A_v^c = -\varepsilon_c I, \quad B_v^c(\alpha) = B^c = I,$$
$$C_v^c(\alpha) = K_i(\alpha), \qquad D_v^c(\alpha) = K_p(\alpha), \tag{27}$$

which is a kind of proportional-plus-integral (PI) control, where $K_i(\alpha)$ is the integral control gain matrix, and $K_p(\alpha)$ is the proportional control gain matrix. Hence the controller for the augmented plant linearization family (22) has the final form

$$\begin{bmatrix} \dot{x}^c \\ v \end{bmatrix} = \begin{bmatrix} -\varepsilon_c I & I & -I \\ K_i(\alpha) & K_p(\alpha) & -K_p(\alpha) \end{bmatrix} \begin{bmatrix} x^c \\ \delta y \\ \delta r \end{bmatrix}, \quad \forall \alpha \in \Omega. \tag{28}$$

The linearized closed-loop system (22) with controller (25) becomes

$$\underbrace{\begin{bmatrix} \delta \dot{x}^p \\ \delta \dot{u} \\ \dot{x}^c \end{bmatrix}}_{\delta \dot{x}} = \underbrace{\begin{bmatrix} A^p(\alpha) & B^p(\alpha) & 0 \\ \eta_c D_v^c(\alpha) C^p(\alpha) & -\eta_c I + D_v^c(\alpha) D^p(\alpha) & \eta_c C_v^c(\alpha) \\ B_v^c(\alpha) C^p(\alpha) & B_v^c(\alpha) D^p(\alpha) & A_v^c(\alpha) \end{bmatrix}}_{A_{cl}(\alpha)}$$
$$\underbrace{\begin{bmatrix} \delta x^p \\ \delta u \\ x^c \end{bmatrix}}_{\delta x} + \underbrace{\begin{bmatrix} 0 \\ -\eta_c D_v^c(\alpha) \\ -B_v^c(\alpha) \end{bmatrix}}_{B_{cl}(\alpha)} \delta r, \quad \forall \alpha \in \Omega.$$
$$\tag{29}$$

For the case where we have plant states as the outputs $\delta y = \delta x^p$, (i.e. $C^p(\alpha) = I, D^p(\alpha) = 0$) the linearized closed-loop system

(22) with controller (28) becomes

$$
\underbrace{\begin{bmatrix} \delta \dot{x}^p \\ \delta \dot{u} \\ \dot{x}^c \end{bmatrix}}_{\delta \dot{x}} = \underbrace{\begin{bmatrix} A^p(\alpha) & B^p(\alpha) & 0 \\ \eta_c K_p(\alpha) & -\eta_c I & \eta_c K_i(\alpha) \\ I & 0 & -\varepsilon_c I \end{bmatrix}}_{A_{cl}(\alpha)} \underbrace{\begin{bmatrix} \delta x^p \\ \delta u \\ x^c \end{bmatrix}}_{\delta x} \\
+ \underbrace{\begin{bmatrix} 0 \\ -\eta_c K_p(\alpha) \\ -I \end{bmatrix}}_{B_{cl}(\alpha)} \delta r, \quad \forall \alpha \in \Omega.
\tag{30}
$$

3 Stability Analysis

In this section we show the stability of the closed-loop nonlinear system by using "global linearization" technique. The stability is due to the existence of a single quadratic Lyapunov function for all $\alpha \in \Omega$, by computing a single Lyapunov matrix P using convex optimization tools.

Assumption 1. *The matrices A_{cl} and B_{cl} are bounded*

$$
||A_{cl}(t)|| \le k_A, \quad ||B_{cl}(t)|| \le k_B, \quad \forall t > 0,
\tag{31}
$$

where k_A and k_B are constants.

To analyze the stability of the nonlinear closed-loop system, we use a technique known as "global linearization" developed in [9–11].

Theorem 1. *Consider the closed-loop system (21), and assume there is a family of equilibrium points (x_e, r_e) such that $F(x_e, r_e) = 0$. Define $A_{cl}^{nl} = \frac{\partial F}{\partial x} \in \overline{S}, \ \forall x \in D_x$, where $\overline{S}$ is the set of linearizations of system (21)*

$$
\overline{S} := \{A_{cl}^{nl}, \forall x \in D_x\}.
\tag{32}
$$

Assume there exists a symmetric positive definite matrix P, such that

$$
P A_{cl}^{nl} + A_{cl}^{nl\mathsf{T}} P < 0, \quad \forall A_{cl}^{nl} \in \overline{S},
\tag{33}
$$

then the system (21) is asymptotically stable.

Remark 1. *In practice we can not obtain $\overline{S}$, instead, we can linearize system (21) for a large number of states x_i, $i = 1, \dots, L$, which we claim is sufficient to cover the set of actual operating conditions, to show the stability of the closed-loop system. Define S as a matrix polytope described by its vertices*

$$
S := \mathrm{Co}\{A_{cl_1}^{nl}, \dots, A_{cl_L}^{nl}\},
\tag{34}
$$

where $A_{cl_i}^{nl} = \frac{\partial F}{\partial x}\big|_{x=x_i} \in S$, for all $i \in \{1, 2, \dots, L\}$. Note that $A_{cl_i}^{nl}$ can be obtained by linearizing the nonlinear system (21) at non-equilibrium points (transient condition), and also at equilibrium points (steady state condition), which in this paper, are represented by $A_{cl}(\alpha_i)$. Then using convex optimization tools [37, 38], we compute a common symmetric positive definite matrix P, such that

$$
P A_{cl_i}^{nl} + A_{cl_i}^{nl\mathsf{T}} P < 0, \quad \forall i \in \{1, 2, \dots, L\}.
\tag{35}
$$

With assumption 1 satisfied, we can verify $A_{cl}(\alpha) \in S$, for all $\alpha \in \Omega$, then system (29) is also asymptotically stable.

4 Turboshaft Engine Example

We apply the proposed output dependent gain scheduling controller to a physics-based model of the JetCat SPT5 turboshaft engine driving a variable pitch propeller developed in [39, 40]. Note that some of the plant states and inputs have been non-dimentionalized by their design values; fuel flow input, u_1, is divided by 0.0035323 (kg/s), core spool speed, N_2, which is the first plant state (x_1^p), and is divided by 170000 RPM, and fan spool speed, N_1, which is the second plant state (x_2^p), and is divided by 7000 RPM. The complete setup of the JetCat SPT5 turboshaft engine with a variable pitch propeller on the test stand is shown in Figure 2.

4.1 Equilibrium Manifold

For a standard day at sea level condition we chose five properly separated equilibrium points on the plant equilibrium manifold for linearizing the plant model at those points. The linearization matrices for these five equilibrium points and steady state values of the engine variables, scheduling parameter and control parameters are given as follows:
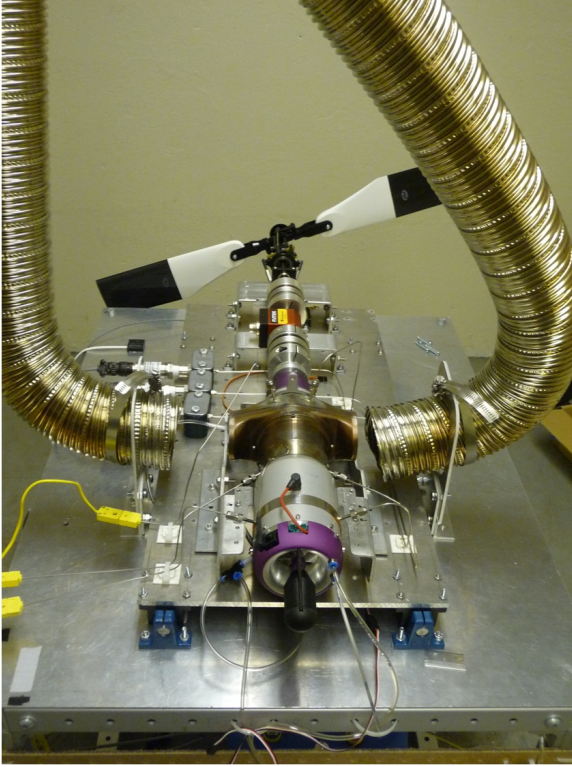
Equilibrium Point 1 (Full Thrust):
$u_{1e1} = 1.0$, $u_{2e1} = 16$ (deg), $x_{1e1} = 1.0$, $x_{2e1} = 0.9524$, $T_{e1} = 255.8685$ (N), $\alpha_1 = 1.3810$, and

$$
A_1^p = \begin{bmatrix} -5 & 0 \\ 3.5 & -2.3 \end{bmatrix}, B_1^p = \begin{bmatrix} 1.4 & 0 \\ 0.63 & -0.085 \end{bmatrix}, C_1^p = I, \\
Ki_1 = \begin{bmatrix} -0.5 & -0.5 \\ -0.5 & -0.5 \end{bmatrix}, Kp_1 = \begin{bmatrix} -0.5 & -0.5 \\ -0.5 & -0.5 \end{bmatrix}.
\tag{36}
$$

Equilibrium Point 2:
$u_{1e2} = 0.7$, $u_{2e2} = 16$ (deg), $x_{1e2} = 0.8826$, $x_{2e2} = $

FIGURE 2. JetCat SPT5 turboshaft engine setup on the test stand

0.6263, $T_{e2} = 181.9711$ (N), $\alpha_2 = 1.0822$, and

$$A_2^p = \begin{bmatrix} -2.83 & -0.0008 \\ 1.20 & -2.10 \end{bmatrix}, B_2^p = \begin{bmatrix} 1.14 & 0 \\ 0.78 & -0.054 \end{bmatrix}, C_2^p = I,$$
$$Ki_2 = \begin{bmatrix} -0.4 & -0.4 \\ -0.4 & -0.4 \end{bmatrix}, Kp_2 = \begin{bmatrix} -0.4 & -0.4 \\ -0.4 & -0.4 \end{bmatrix}. \tag{37}$$

Equilibrium Point 3 (Cruise):
$u_{1e3} = 0.4685$, $u_{2e3} = 16$ (deg), $x_{1e3} = 0.7264$, $x_{2e3} = 0.5$, $T_{e3} = 70.5125$ (N), $\alpha_3 = 0.8818$, and

$$A_3^p = \begin{bmatrix} -1.9 & 0.061 \\ 0.45 & -1.1 \end{bmatrix}, B_3^p = \begin{bmatrix} 1.57 & 0 \\ 0.3 & -0.023 \end{bmatrix}, C_3^p = I,$$
$$Ki_3 = \begin{bmatrix} -0.3 & -0.3 \\ -0.3 & -0.3 \end{bmatrix}, Kp_3 = \begin{bmatrix} -0.3 & -0.3 \\ -0.3 & -0.3 \end{bmatrix}. \tag{38}$$

Equilibrium Point 4:
$u_{1e4} = 0.3$, $u_{2e4} = 16$ (deg), $x_{1e4} = 0.5327$, $x_{2e4} = $

0.3678, $T_{e4} = 38.155$ (N), $\alpha_4 = 0.6473$, and

$$A_4^p = \begin{bmatrix} -0.85 & 0.032 \\ 0.32 & -0.64 \end{bmatrix}, B_4^p = \begin{bmatrix} 1.1 & 0 \\ 0.17 & -0.011 \end{bmatrix}, C_4^p = I,$$
$$Ki_4 = \begin{bmatrix} -0.2 & -0.2 \\ -0.2 & -0.2 \end{bmatrix}, Kp_4 = \begin{bmatrix} -0.2 & -0.2 \\ -0.2 & -0.2 \end{bmatrix}. \tag{39}$$

Equilibrium Point 5 (Idle):
$u_{1e5} = 0.145$, $u_{2e5} = 16$ (deg), $x_{1e5} = 0.295$, $x_{2e5} = 0.161$, $T_{e5} = 7.317$ (N), $\alpha_5 = 0.3361$, and

$$A_5^p = \begin{bmatrix} -0.38 & -0.0008 \\ 0.26 & -0.34 \end{bmatrix}, B_5^p = \begin{bmatrix} 0.7 & 0 \\ 0.1 & -0.0024 \end{bmatrix}, C_5^p = I,$$
$$Ki_5 = \begin{bmatrix} -0.1 & -0.1 \\ -0.1 & -0.1 \end{bmatrix}, Kp_5 = \begin{bmatrix} -0.1 & -0.1 \\ -0.1 & -0.1 \end{bmatrix}. \tag{40}$$
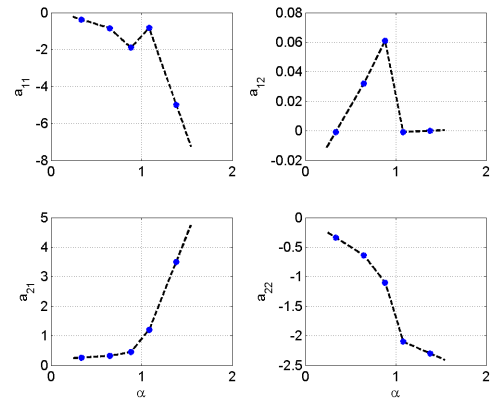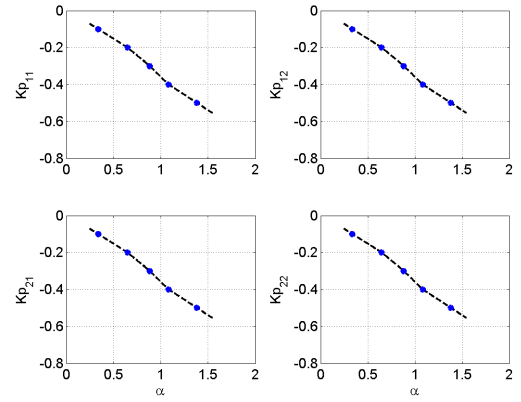

FIGURE 3. $A^p(\alpha)$ components as functions of parameter α

Other controller parameters are $\varepsilon_c = 1$, $\eta_c = 3$. The elements of $A^p(\alpha)$ and $B^p(\alpha)$ matrices have been shown as functions of scheduling parameter α in figures 3 and 4. In this simulation, the scheduling parameter, α, is defined to be the Euclidean norm of the gas turbine engine spool speeds, which are the plant outputs and capture the engine nonlinearities. Piecewise linear interpolation has been used to compute matrices in between the available linearization matrices of each pair of adjacent equilibrium points. The equilibrium values of the plant states and control inputs are shown in figure 5 as functions of scheduling parameter α. Piecewise linear interpolation has been used to compute equilibrium values in between each pair of adjacent equilibrium points. The equilibrium manifold in a 3D space of two spool speeds and fuel flow control input is shown in figure 6. The elements of control matrices $K_p(\alpha)$ and $K_i(\alpha)$ have
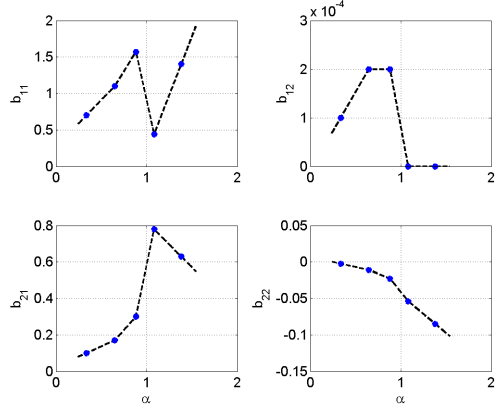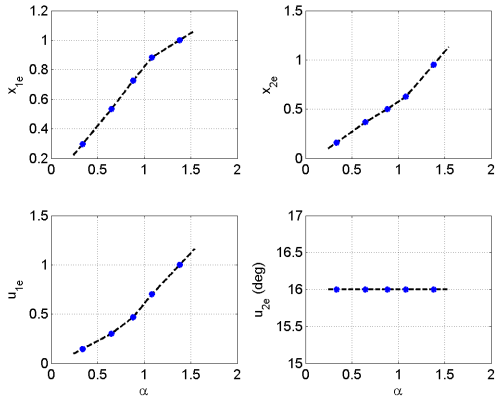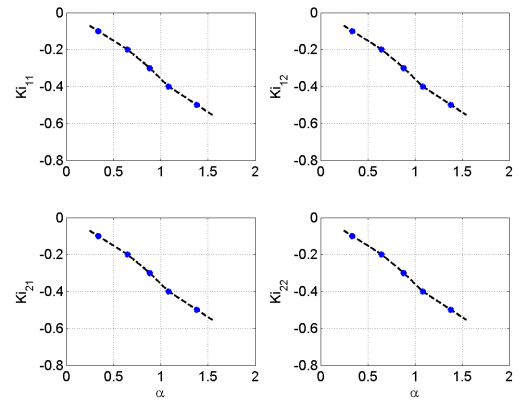
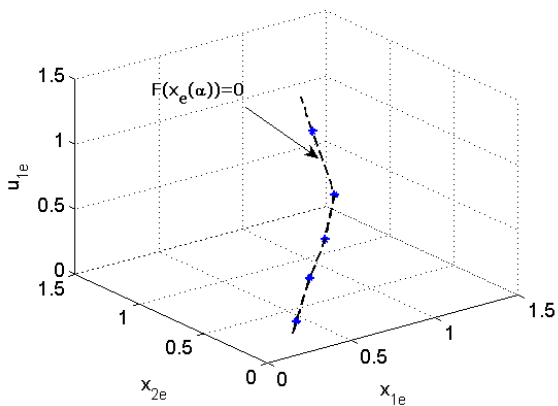**FIGURE 4**. $B^p(\alpha)$ components as functions of parameter α

FIGURE 5. $x_e^p(\alpha)$ and $u_e(\alpha)$ as functions of parameter α

FIGURE 6. Engine equilibrium manifold in 3D space of spool speeds and fuel control input

**FIGURE 7**. $K_p(\alpha)$ components as functions of parameter α

FIGURE 8. $K_i(\alpha)$ components as functions of parameter α

been shown as functions of scheduling parameter α in figures 7 and 8. Piecewise linear interpolation has been used to interpolate $K_p(\alpha)$ and $K_i(\alpha)$ using the predesigned indexed linear controllers, which are given in equations (36) to (40).

4.2 Simulation Results

To show the stability of the closed-loop system, 40 different (30 equilibrium, and 10 non-equilibrium) linearizations have been used, to solve inequality (35), in Matlab with the aid of YALMIP [37] and SeDuMi [38] packages. The numerical value for the common matrix P is

$$P = \begin{bmatrix} 0.5232 & 0.0059 & 0.0913 & -0.0177 & -0.0293 & -0.0011 \\ 0.0059 & 0.3406 & 0.0132 & -0.0082 & -0.0862 & -0.0114 \\ 0.0913 & 0.0132 & 0.1721 & -0.0461 & 0.0044 & 0.0105 \\ -0.0177 & -0.0082 & -0.0461 & 0.1275 & 0.0388 & 0.0282 \\ -0.0293 & -0.0862 & 0.0044 & 0.0388 & 0.2684 & -0.0211 \\ -0.0011 & -0.0114 & 0.0105 & 0.0282 & -0.0211 & 0.2484 \end{bmatrix},$$

where its condition number is 6.8910. Here, we implement the
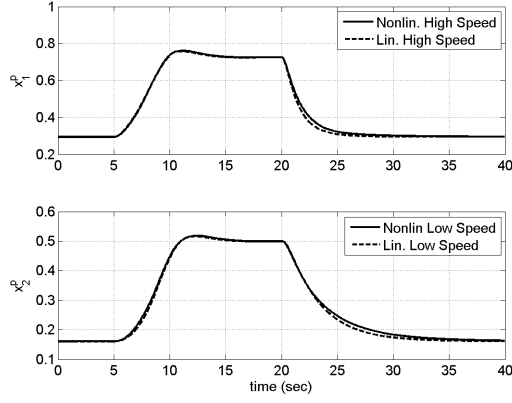
FIGURE 9. History of the states ($x^p(t)$) for the nonlinear system and the linear parameter dependent model
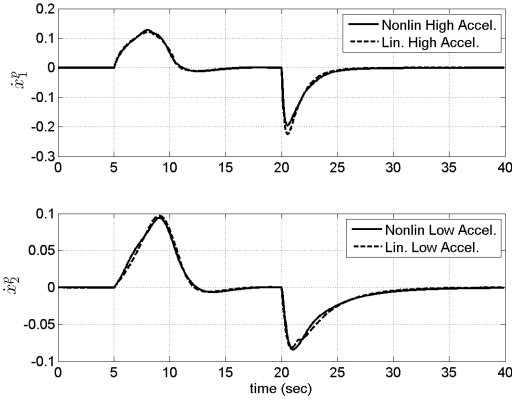
FIGURE 10. History of the rate of states ($\dot{x}^p(t)$) for the nonlinear system and the linear parameter dependent model

proposed parameter dependent gain scheduled controller to operate the JetCat SPT5 turboshaft engine. This case study, simulates the engine acceleration from the idle thrust to the cruise condition and then its deceleration back to the idle condition in a stable manner, with proper tracking performance, for the standard day sea level condition. Simulation results are shown in figures 9 to 25. Figure 9 and 10, shows the history of the nonlinear system and the linear parameter dependent model states, $x^p(t)$, and the rate of states, $\dot{x}^p(t)$. We can conclude that, the linearized model is a very good approximation of the real nonlinear plant. Figure 11, shows the history of the norm of the closed-loop system
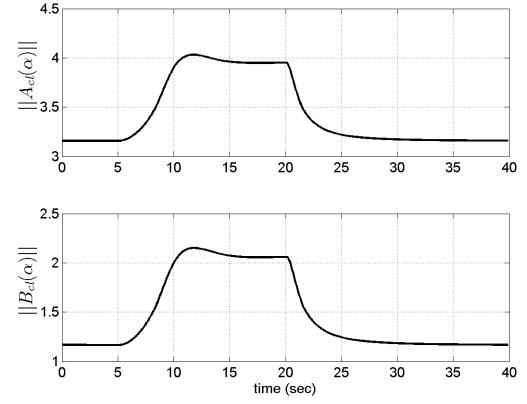

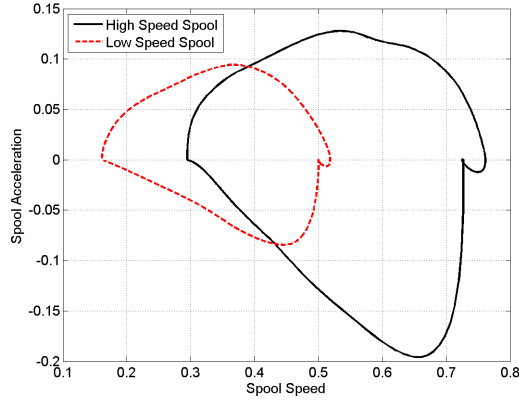FIGURE 11. Norm of the closed-loop system matrices ($||A_{cl}||$), and ($||B_{cl}||$)
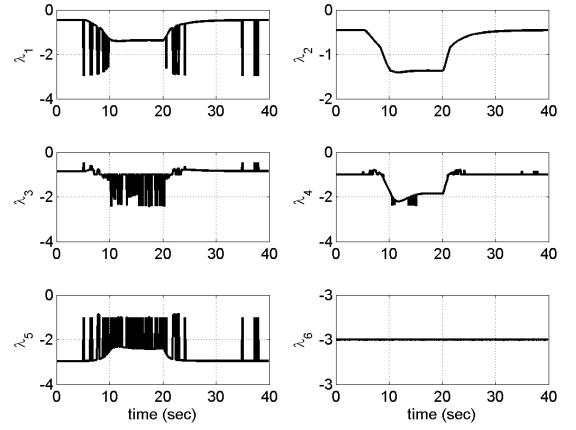
FIGURE 12. Closed-loop system eigenvalues ($\lambda[A_{cl}(\alpha)]$)

matrices $||A_{cl}||$, and $||B_{cl}||$. The figure shows the boundedness of these two matrices, in accordance with Assumption 1, where $k_A = 4.0327$, and $k_B = 2.1512$. Figure 12, shows the history of the closed-loop system matrix eigenvalues $\lambda\{A_{cl}\}$. All the eigenvalues remain negative with the time change of the scheduling parameter α. Figure 13, shows the history of the scheduling parameter $\alpha = p(y) = ||y|| = ||x^p||$ (the Euclidean norm of the engine spool speeds). The history of the scheduling parameter rate $\dot{\alpha} = \frac{x^{p^\top}\dot{x}^p}{||x^p||}$, also has been plotted. Both α and $\dot{\alpha}$ are bounded. Figure 14, shows the phase plot for both spool dynamics. Figure 15, shows the evolution of the plant states which are high and low spool speeds. Figure 16, shows the time evolution of the controller states.

Figures 17 and 18, show the outputs (i.e., high and low spool speeds) tracking their reference signals properly. Figure
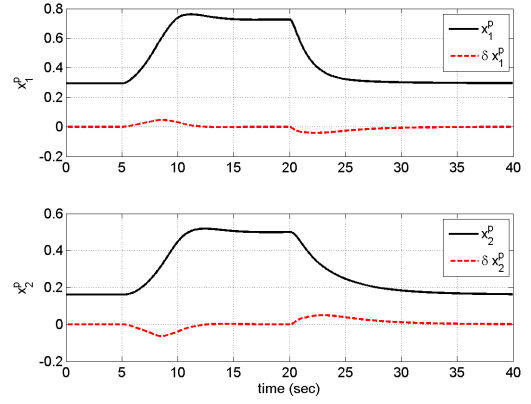
9

Copyright © 2013 by ASME
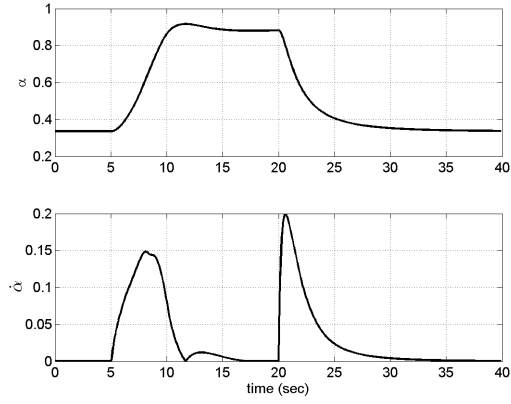
FIGURE 13. Scheduling Parameter ($\alpha = ||x^p||$) and its rate of change ($\dot{\alpha} = \frac{x^{p\top}\dot{x}^p}{||x^p||}$)

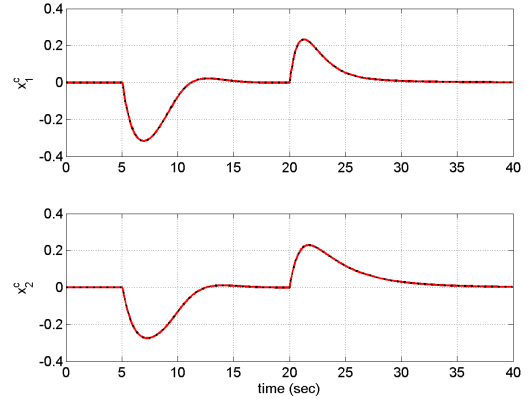**FIGURE 15.** Plant states: high and low spool speeds (x^p)

FIGURE 14. High and low spool speeds vs. high and low spool accelerations
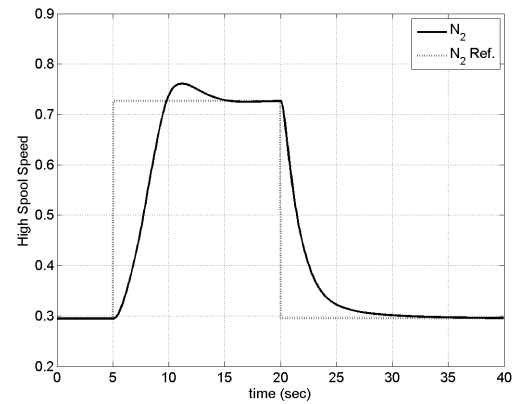

FIGURE 16. Controller states (x^c)

19, shows the history of the thrust and it is following its reference command from idle to cruise condition and then back to the idle for standard day, sea level condition. Figure 20, shows the evolution of the control inputs $v(t) = [v_1(t), v_2(t)]^T$, which are inputs to the augmented system, each element is corresponding to one of the control inputs to the original system. Figure 21, shows time rates of fuel and prop pitch angle inputs. Figure 22, shows fuel flow and propeller pitch angle histories as the control inputs to the plant. Figures 23 and 24, show the evolution of the controllers integral ($K_i(\alpha)$) and proportional ($K_p(\alpha)$) gain matrices. These gains have been obtained by interpolation using the predesigned indexed family of fixed-gain controllers, and each controller corresponds to one equilibrium point of the engine. The numerical values of these gains are given in equations (38) to (40), which represents the controller gains for idle and cruise condition and one more equilibrium point in between these two
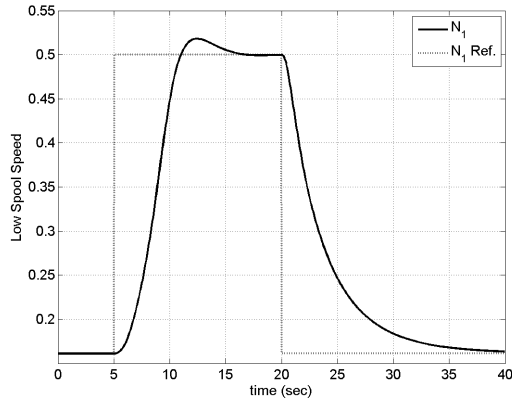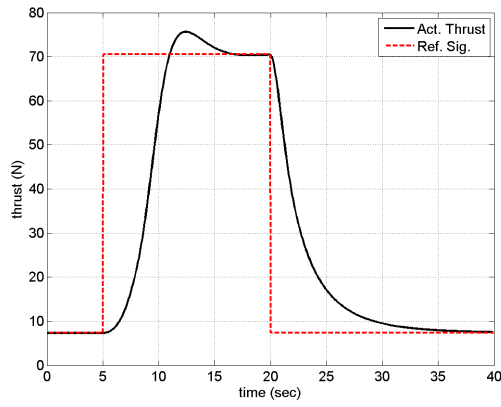

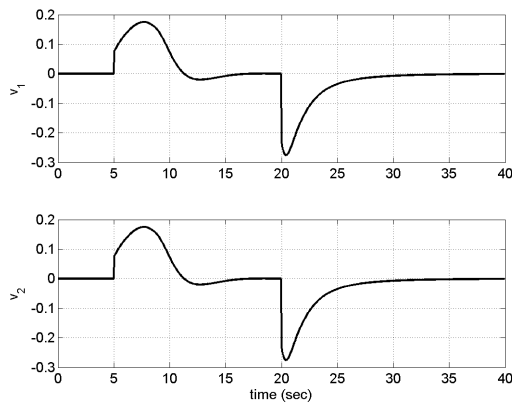**FIGURE 17.** Output: high spool speed and its reference signal

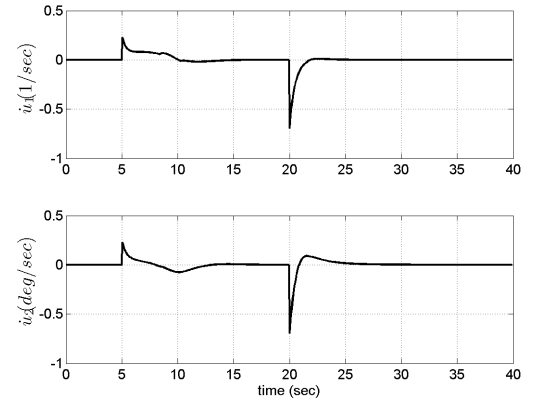operating points. Figure 25, shows the histories of turbine tem-

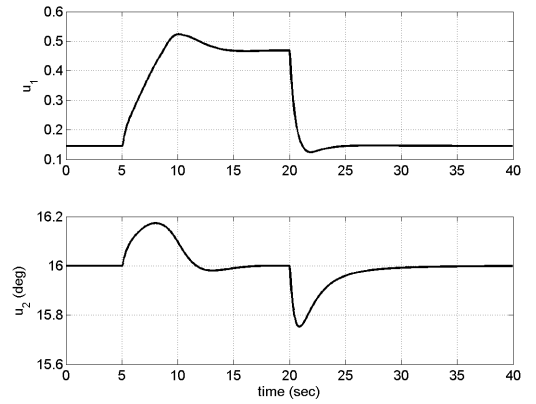**FIGURE 18**. Output: low spool speed and its reference signal

FIGURE 19. Thrust and its reference signal

FIGURE 20. Control inputs to the augmented system ($v(t)$)

**FIGURE 21**. Rate of change for fuel and prop pitch angle control inputs ($\dot{u}(t)$)

FIGURE 22. Fuel and prop pitch angle control inputs ($u(t)$)

pressure ratio and corrected air flow rate.

4.3 Engine Limit Control

To handle the limits on the turbine engine system states and control inputs, the developed gain scheduled control system can be integrated with a reference governor. Reference governors have been developed previously; one of the good examples of this approach is presented in [41]. This method addresses the problem of satisfying input and/or state hard constraints in nonlinear control systems. The approach uses receding horizon strategy and consists of adding to the primal compensated nonlinear system a reference governor. The proposed reference governor is a discrete-time device which handles the reference to be tracked in an on-line fashion. The resulting hybrid system satisfies the constraints as well as stability and tracking requirements [41].

perature, thrust specific fuel consumption (TSFC), compressor

11
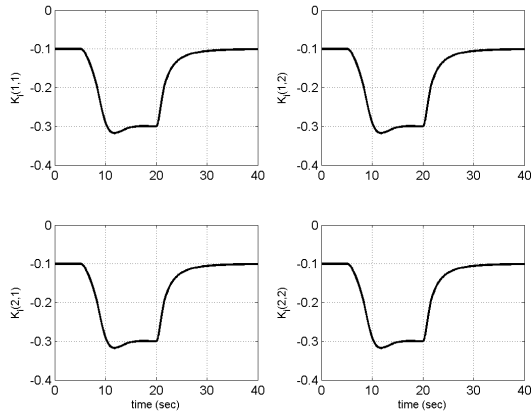
FIGURE 23. Controllers integral gain matrix ($K_i(\alpha)$) elements histories
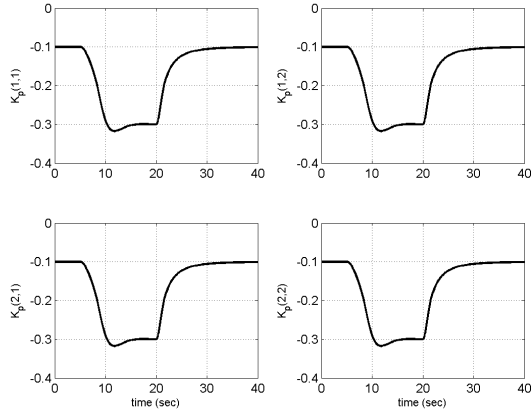
FIGURE 24. Controllers proportional gain matrix ($K_p(\alpha)$) elements histories

5 Conclusions

A MIMO linear parameter dependent model of the nonlinear gas turbine engine system has been developed; and a gain scheduling controller with stability guarantees for this system has been designed. Piecewise linear interpolation technique has been used for interpolating the parameter varying gain scheduling controller in between the predesigned indexed family of fixed-gain controllers. The scheduling variable in the design process is an endogenous parameter (i.e., a function of the plant outputs) and it has been defined to be the Euclidean norm of the gas turbine engine spool speeds. Stability of the closed-loop gas turbine engine system with a gain scheduling controller, has been shown using global linearization method. It also has been shown that a single quadratic Lyapunov function can be computed for this system using convex optimization tools, which guarantees the stability of
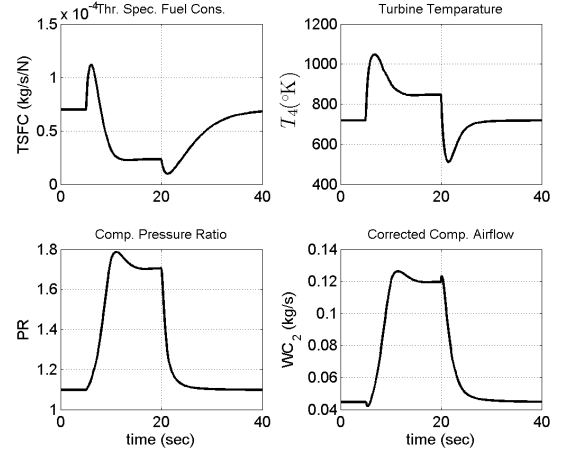
FIGURE 25. Turbine temperature, TSFC, compressor overall pressure ratio and air flow rate histories

the closed-loop nonlinear gas turbine engine system with a gain scheduling controller. Simulation results confirmed the applicability of the proposed controller to the nonlinear physics-based JetCat SPT5 turboshaft engine model for large transients.

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